BASEMENT

ON THE ASSIGNMENT OF LIABILITY:

THE UNIFORM CASE

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1. Introduction

Economists recommend that, in externality situations, agents should pay the marginal social costs of their actions. This is not always a very helpful prescription. Sometimes only simpler ways of shifting costs among agents are to be considered. Generally such simple shifting would not result in the competitive equilibrium being a Pareto optimum. It is natural to ask whether the equilibrium with shifted costs is more efficient than the one when costs are not shifted. Specifically, we seek sets of conditions that are sufficient to identify the more efficient equilibrium.

One situation subject to examination by a model of this sort arises when there is the possibility of an accident between an agent engaged in one activity (a railroad, say) and one engaged in a different activity (a truck, say, or horse drawn wagon). The question we pose is then whether it is more efficient to have accident costs remain on the parties suffering these costs at the time of an accident or to have one of the parties bear a given portion of the costs that have fallen on the other party. This question has been addressed in the legal literature as that of the assignment of liability for accident costs.

Thus we are considering the issue of whether shifting a particular class of costs, pain and suffering of truck drivers for example, will increase efficiency. Our attention has been brought to this question by the interesting book/The Cost of Accidents[1] by Guido Calabresi. He argues that liability ought to be placed so as to

produce the more efficient equilibrium, and calls the person who should bear liability under this rule the "cheapest cost avoider."[2] This paper, then, explores some sets of conditions which, if satisfied, are sufficient for identification of the cheapest cost avoider.[3]

There are many rules for deciding whether to shift costs in a particular accident, other than deciding before the accident that certain costs will be shifted in all accidents involving parties engaged in the two activities.[4] In particular the impact of negligence rules on allocative efficiency has been examined by several authors recently.[5] With any rule, there will be an administrative cost to offset any efficiency gain arising from application of the rule. Calabresi has argued that, particularly in the case of automobile accidents, the negligence rule has become so expensive to administer that it would be preferable to use a strict liability[6] rule, like the one we examine here.

Calabresi has identified three types of costs associated with accidents. Tertiary costs are those that come from the administration of any legal/compensatory scheme to deal with people who have had accidents. Since the alternatives we shall consider do not differ too greatly in their complexity, we shall ignore tertiary costs, on the assumption that they are approximately equal for the two alternatives whose efficiency we compare. Secondary costs are additional costs arising from failure to deal promptly and adequately with the consequences of accidents. They include complications arising from delayed medical treatment and the costs coming from changing marginal utilities of income in the presence of risk aversion and imperfect insurance. We shall also ignore secondary costs, assuming that all

utility functions are linear in income available to spend on other goods. This assumption significantly limits the applicability of these results, perhaps only to accidents between firms and possibly thoroughly insured individuals. <u>Primary costs</u> are the resource costs arising from accidents plus the resources used to avoid them. We do not ignore these costs.

In considering the cheapest cost avoider, Calabresi paid particular attention to two important elements. One is that incentives to avoid accidents can be greatly dissipated by insurance which shifts accident costs from a party without relating the premium he pays to his behaviour. Calabresi calls this case externalization by transfer.[7] As an extreme example /a blanket insurance policy that does not even enquire whether an individual is engaged in some activity giving rise to increased risk. Provided the other potential parties to accidents have not had their incentives similarly blunted, there is an allocative gain from shifting costs to the person who will have his incentives affected.[8] A second element Calabresi has considered is the frequent need to alter behaviour of other parties. Some individuals or groups may be in a better position to do this than others. He calls the group in the more favourable position to affect behaviour of others, the best briber. We shall be considering models in which these issues do not arise.[9]

We shall focus on two issues that are relevant for identification of the cheapest cost avoider - the relative importance of accident avoidance measures by the parties engaged in the different activities, and the response of avoidance to incentives. After presenting the basic model in the next section, we consider the special case where

the only externalities are in the costs for which liability is to be assigned. In this case, greater efficiency is achieved by placing liability on the party whose accident avoidance expenditures, above the minimum of self protection, are more important. This case is special because some externalities generally fall outside the parties to the accident (on the government for example) and because it is not generally practicable to identify all externality costs accurately. (For example, some might arise outside an accident context.) We then proceed to consider the more complicated and interesting cases. For this paper we assume that everyone engaged in a particular activity is identical and we assume large numbers of such individuals with no externalities between agents engaged in the same activity. In later papers we plan to approach models with large numbers, but without the other two assumptions.

2. Basic Model

We assume that there are two activities that interact. In each activity there are a large (not necessarily equal) number of participants. The participants in each activity are identical, but the two types engaged in the two activities may differ. No one is engaged in both activities. Let us denote by x the level of accident avoidance of each participant in activity one. This might represent actual expenditures on safety equipment or efforts, like paying attention, which affect accident probabilities. It might represent a choice made for non-safety reasons which affects safety like the increase of the speed or number of miles driven. The variable x might simply represent potentially damaged items which are not subjected to the

risk of damage. Similarly we denote by y the level of accident avoidance of each participant in activity two. We denote the total expected costs, net of the utility from engaging in the activity, of participants in activity one by A(x,y). On the assumption that there are no externalities within an activity, the care level x is chosen to minimize A given y. $x'(y)$ is defined as the optimal level of care:

$$x'(y) \quad \text{minimizes} \quad A(x,y). \tag{1}$$

From the large numbers assumption, we model individual choice with the decisions of others taken as given.

Similarly, we denote by $\tilde{B}(x,y)$ the total expected accident costs and accident avoidance costs (net of utility) of participants in activity two. Their optimal level of care is denoted by $y*(x)$:

$$y*(x) \quad \text{minimizes} \quad \tilde{B}(x,y). \tag{2}$$

There is competitive equilibrium with correct perceptions when the optimizing choices are predicted on a correct perception of the simultaneous choices being made in the other activity. That is, $x',y*$ is a competitive equilibrium if

$$x' = x'(y*)$$
$$y* = y*(x'). \tag{3}$$

We assume throughout the paper that there exists a unique competitive equilibrium for any assignment of costs.

Transferred Costs

Now assume that there are some costs which are to be transferred from participants in activity two to the participants in activity one. Denote the expected value of these costs by $C(x,y)$. Those in activity one now seek to minimize $A(x,y) + C(x,y)$:

$$x^*(y) \quad \text{minimizes} \quad A(x,y) + C(x,y). \tag{4}$$

The costs of activity two are reduced by $C(x,y)$, and are now

$$B(x,y) = \tilde{B}(x,y) - C(x,y). \tag{5}$$

The choice in activity two is described by $y'(x)$:

$$y'(x) \quad \text{minimizes} \quad B(x,y). \tag{6}$$

We have a new competitive equilibrium with transferred costs, that is, with liability assigned to activity one, at the values x^*,y' satisfying

$$x^* = x^*(y')$$
$$\tag{7}$$
$$y' = y'(x^*).$$

There are two equilibria to compare: (x^*,y') with liability assigned to activity one, and (x',y^*) with liability assigned to activity two. We are interested in discovering which equilibrium has lower aggregate costs of accidents and accident avoidance. In addition to the costs A, B, and C that fall on participants in the two activities, one should consider also expected costs $D(x,y)$ that fall outside the two activities. For example, they might fall on the government or on passersby. They could arise from misperceived costs,

provided all participants misperceive in the same way. We define the sum of costs S, by

$$S(x,y) = A(x,y) + B(x,y) + C(x,y) + D(x,y). \tag{8}$$

The problem at hand is to find sufficient conditions for comparing $S(x^*,y')$ with $S(x',y^*)$ and thus to judge which assignment of liability is more efficient.

3. All Externalities Transferable

We start with the simple case where all externality costs can be assigned to one of the activities. That is, we assume that A is a function just of x, B is a function just of y and D is equal to zero. We assume that A and B are convex and continuously differentiable. Given these assumptions, the values x' and y' that minimize $A(x)$ and $B(y)$ respectively do not depend on the choice of care in the other activity. Thus they serve as natural origins, being the level of care for self protection. We will say that <u>care in activity one is more effective</u> if for any (positive) amount of care above the self protection level it is more efficient to have that care taken in activity one,[10] that is

$$\text{For } t > 0, \quad S(x' + t, y') \leq S(x', y' + t). \tag{9}$$

We can now state the result for this simple case

<u>Theorem 1</u> With costs $A(x)$, $B(y)$, $C(x,y)$, and with $D = 0$, which are convex and continuously differentiable, it is more efficient to have liability placed on activity one, if activity one is

more effective in cost avoidance ((9) is satisfied) and we are dealing with external economies:[11]

$$C_2(x',y') < 0 \qquad (10)$$

where C_2 is the partial derivative of C with respect to its second argument.

Proof: Let $t \equiv y^*(x') - y'$. Since $B(y) + C(x',y)$ is decreasing in y at y' (and convex), the minimizing level y^* exceeds y'. Thus t is positive and we can use (9) to conclude that

$$S(x',y^*) = S(x',y' + (y^* - y')) \overset{\geq}{} S(x' + y^* - y',y'). \qquad (11)$$

Since $x^*(y')$ minimizes $A(x) + C(x,y')$ (and so minimizes $S(x,y')$) we have

$$S(x' + y^* - y',y') \overset{\geq}{} S(x^*,y'). \qquad (12)$$

This completes the proof.//

While this result does little more than confirm the relationship between effectiveness (as defined in (9)) and efficiency, it is interesting how fragile the result is. Making A or B depend on care in the other activity or permitting D to be non zero requires additional substantive assumptions to preserve the ability to identify the cheapest cost avoider. We shall see this as we examine next the case where D is not zero.

4. Outside Externalities

While preserving the assumptions that A depends only on x and that B depends only on y, dropping the assumption that D is zero ends the validity of Theorem 1 without further assumptions. This is true despite the fact that effectiveness is defined in terms of total costs, including the costs falling outside the two activities. The proof, itself, breaks down because $x*(y')$ does not generally optimize $S(x,y')$ even though it optimizes $A(x) + B(y') + C(x,y')$.

Given that we are dealing with the case of external economies it seems natural to add the assumption that $D(x,y)$ is decreasing in x. Even this assumption is not enough, for $x* - x'$ might well be less than $y* - y'$ and so $D(x*,y')$ greater than $D(x' + y* - y',y')$.[12]

This is the familiar difference between the total and marginal valuations. The relative sizes of $x* - x'$ and $y* - y'$ depend on the values of the derivatives of A, B, and C while the assumptions entertained thus far consider only the level of total costs. Thus to complete an argument for this case, we add the assumption that

For $t > 0$, $A'(x' + t) + C_1(x' + t,y') \leq B'(y' + t) + C_2(x',y' + t)$. (13)

where A' and B' are derivatives.

To interpret (13), consider an imaginary market for care, with shadow price q. If $\bar{x}(q)$ minimizes $A(x) + C(x,y') + qx$, and $\bar{y}(q)$ minimizes $B(y) + C(x',y) + qy$, (14) is equivalent to

$$\bar{x}(q) - x' \overset{>}{-} \bar{y}(q) - y' \quad \text{for all} \quad q \tag{14}$$

$$\text{such that} \quad \bar{y}(q) \overset{>}{-} y'.$$

To see this, put $t = \bar{y}(q) - y'$ in (13). We have

$$A'(x' + \bar{y}(q) - y') + C_1(x' + \bar{y}(q) - y', y') \overset{<}{-} - q, \tag{15}$$

which implies (by convexity) that costs would be increased if x were reduced, i.e. $\bar{x}(q) \overset{>}{-} x' + \bar{y}(q) - y'$, which is (14). Because of (14), we shall describe assumption (13) by saying that activity one has a greater excess supply of care when liable.

In particular (14) holds for $q = 0$, when $\bar{x}(q) = x^*$, $\bar{y}(q) = y^*$, so that

$$x^* - x' \overset{>}{-} y^* - y'. \tag{16}$$

This condition and the assumed decrease in D with increases in x completes the necessary argument. We state this formally.

Theorem 2   With costs $A(x)$, $B(y)$, $C(x,y)$, $D(x,y)$ which are convex and continuously differentiable, it is more efficient to have liability placed on activity one if activity one is more effective in cost avoidance ((9) is satisfied), activity one has a greater excess supply of care when liable ((13) is satisfied) and we are dealing with external economies so that

$$C_2(x',y') < 0, \tag{17}$$

$$D_1(x,y') < 0 \quad \text{for} \quad x > x'.$$

Proof: As in Theorem 1, $y^* - y' > 0$. Therefore, by (9)

$$S(x',y^*) = S(x',y' + y^* - y') \geq S(x' + y^* - y',y')$$

$$= A(x' + y^* - y',y') + B(y') + C(x' + y^* - y',y') \quad (18)$$

$$+ D(x' + y^* - y',y').$$

Since x^* optimizes $A(x) + B(y') + C(x,y')$, and since $x^* > x' + y^* - y'$ and D is decreasing in its first argument, the last expression in (18) is at least as large as $S(x^*,y')$. //

5. Nontransferable Externalities

When expected costs depend in part on the level of care in the other activity, whichever way liability is assigned, we find it necessary to impose much more severe restrictions if we are to be able to identify the more efficient assignment. In our previous arguments, we were using (x',y') as a fixed origin. This was quite natural, since x' was determined by A, y' by B. In the more general case, (x',y^*) are determined simultaneously, as are (x^*,y'). This will make it harder to decide whether shifting liability increases the equilibrium level of care in the activity to which liability has been shifted; and whether the shift has a greater effect on care in one activity than in the other. In order to isolate considerations of these issues, we state two Lemmas relating efficiency to equilibrium quantities. Then, we will consider sufficient conditions to satisfy the restrictions on individual quantities.

As before, functions are assumed continuously differentiable and convex in the control variables. In the present section, $D = 0$; the

adaptation of these results for a non zero D will be discussed in the next section.

Lemma 1   It is more efficient to have liability placed on activity one if (i) activity one is more effective in cost avoidance (if (9) is satisfied), and $y* \geq y'$; and (ii) B is a nonincreasing function of x, and $x* - x' \geq y* - y'$.

Variants of Lemma 1   The Lemma remains true if assumptions (i) are replaced by the assumptions that $y* \leq y'$ and care reduction in activity one is more effective / than that in activity two, in the sense that

$$S(x' - t, y') \leq S(x', y' - t) \quad \text{whenever} \quad t > 0. \tag{19}$$

The Lemma also remains true if assumptions (ii) are replaced by the assumptions that B is a nondecreasing function of x, and $x* - x' \leq y* - y'$. These assertions may be readily checked by the same method of proof as for the stated Lemma:

Proof of Lemma 1   We have to show that $S(x*, y') \leq S(x', y*)$. By assumptions (i),

$$S(x', y*) \geq S(x' + y* - y', y')$$

$$= \left[A(x' + y* - y', y') + C(x' + y* - y', y') \right] + B(x' + y* - y', y')$$

$$\geq \left[A(x*, y') + C(x*, y') \right] + B(x*, y'),$$

because $x*$ minimizes $A(x, y') + C(x, y')$, and from assumptions (ii).//

This Lemma follows very closely the structure of the argument in Theorem 2.

The second Lemma is different in character from the earlier results, but uses a very simple argument.

Lemma 2   It is more efficient to have liability placed on activity one
    if

        (i) $A(x',y) + C(x',y)$ is nondecreasing in y and $y* \geq y'$,

    and (ii) $B(x,y*)$ is nonincreasing in x and $x* \geq x'$.

Variants of Lemma 2   The conclusion is unaffected if (i) is replaced
by the assumptions that $y* \leq y'$ and $A + C$ is nonincreasing in
y.   A similar change in (ii) also leaves the conclusion unaffected.

Proof of Lemma 2

$$S(x',y*) = \left[A(x',y*) + C(x',y*)\right] + B(x',y*)$$

$$\geq \left[A(x',y') + C(x',y')\right] + B(x*,y*), \quad \text{by (i) and (ii)}$$

$$\geq \left[A(x*,y') + C(x*,y')\right] + B(x*,y'),$$

because x* minimizes $A + C$ when $y = y'$, and y' minimizes B
when $x = x*$.   Therefore $S(x',y*) \geq S(x*,y').$//

It is interesting to notice that, unlike those of Lemma 1, the conditions of Lemma 2 are independent of the way in which the care variables are measured since no comparison is made between the two activities. However assumption (i) seems rather unnatural for the applications we have in mind. It says that increased care by people in the second activity increases the costs of those engaged in activity one when liability is on activity one. This could happen where care affects the allocation of costs (by courts or otherwise) more than the level of total costs.[13]

A third Lemma, which exploits assumptions about the asymmetry of S relative to zero levels of care rather than (x',y'), is discussed in the Appendix.

In order to apply the Lemmas, we have to consider how the equilibrium care levels can be related under various assumptions about A, B, and C. The situation can be conveniently portrayed in an (x,y)-diagram, but there are in principle a great many different cases to be considered. One case is shown in Figure 1. There the curves defined by the functions $x^*(y)$, $x'(y)$, $y^*(x)$, $y'(x)$ are shown, the two points representing equilibria being ringed. In this case, the curves are all decreasing from left to right, i.e. the reaction functions are all decreasing: more care in one activity induces a decrease in care in the other. In addition, the x-curves decline more steeply than the y-curves.

It is natural to consider this a stable case. The sequence of points obtained by having the two activities respond to one another successively converges to the equilibrium, as shown. A sufficient condition for stability more generally is that

$$\left|\frac{dx^*}{dy}\right| \left|\frac{dy'}{dx}\right| < 1, \qquad \left|\frac{dx'}{dy}\right| \left|\frac{dy^*}{dx}\right| < 1. \qquad (19)$$

In words, the absolute slope of the y-curves should be less than the absolute slope of the x-curves, as seen in the diagrams. We shall consider only examples with this property.

The case shown in Figure 1 has the x^*-curve to the right of the x'-curve, and the y^*-curve above the y'-curve. These properties follow immediately from the assumptions that C is a decreasing function of both x and y. Cases where the derivatives of C are

single-signed are the most readily interpretable, and we shall therefore
not consider cases where the x-curves, or the y-curves, intersect one
another.

Pursuing this particular case, we see that

$$x* \overset{>}{=} x', \qquad y* \overset{>}{=} y'. \tag{21}$$

Thus Lemma 2 is immediately applicable. Lemma 1 is applicable only if
we can determine an inequality between $x* - x'$ and $y* - y'$. There
are several ways of doing so:

(I) If the x'-curve slopes downwards less steeply than the forty-five
degree line, the line joining the two equilibria, lying above the
x'-curve, certainly has an absolute slope less than forty-five degrees.
Therefore $x* - x' \overset{>}{=} y* - y'$. The same is true if instead of x', the
$x*$-curve slopes downwards at less than forty-five degrees. The slopes
are the inverse of dx/dy, whose absolute values are, in terms of A
and C, A_{12}/A_{11} and $(A_{12} + C_{12})/(A_{11} + C_{11})$ respectively. It is
sufficient for the applicability of Lemma 1 that one of these always
exceeds unity. We shall call the absolute values of dx'/dy and
$dx*/dy$ the _nonliable_ and _liable responses_ of activity one (with
similar definitions for activity two); and refer to the sign of the
derivatives as a _positive_ or _negative reaction_. To satisfy (ii) of
Lemma 1, our condition is that the response of activity one under one
regime or the other be not less than one[14] and that $B_1 \overset{<}{=} 0$.

(II) Another condition that implies $x* - x' \overset{>}{=} y* - y'$ is the 'supply
condition' discussed in section 4 as equation (13):

For $t > 0$, $A_1(x' + t, y') + C_1(x' + t, y') \overset{<}{=} B_2(x', y' + t) + C_2(x', y' + t)$. (22)

The argument is the same as in section 4. Since $B_2 + C_2$ vanishes at $(x',y*)$, (22) implies that $A_1 + C_1 < 0$ at $(x' + y* - y',y')$. $A + C$ being a convex function of x, which attains a unique minimum on the line (x,y') when $x = x*$, $A_1 + C_1 > 0$ when $x > x*$. Therefore $x' + y* - y' \overset{<}{=} x*$, as claimed.

A similar argument, starting from the point $(x*,y*)$ rather than the point (x',y') shows that the condition

$$t > 0 \rightarrow A_1(x* - t,y*) \overset{>}{=} B_2(x*,y* - t) \qquad (23)$$

implies that $x* - x' \overset{>}{=} y* - y'$.

These conditions are quite different from one another, and it is hard to see how to subsume them under a single more general condition. Notice that condition (22), when combined with a further assumption that

$$\text{For } t > 0, \quad B_1(x' + t,y') \overset{<}{=} A_2(x',y' + t), \qquad (24)$$

implies that activity one is the more effective in cost avoidance (in the sense of (9)). Adding (22) and (24), we have

$$t > 0 \rightarrow S_1(x' + t,y') \overset{<}{=} S_2(x',y' + t). \qquad (25)$$

Integration of inequality (25) between 0 and a positive value of t yields inequality (9). Since we are dealing with a case where $B_1 \overset{<}{=} 0$, (24) can be interpreted as saying that activity two has more to gain on the margin from the external effects of care (when nonliable) than has activity one. Similarly, (22) says that own care is more effective on the margin in activity one (when liable) than in activity two. (23) makes

a similar statement about the activities in the absence of liability.

If we bring together our various conditions, we have the following theorem:

Theorem 3   Let C be a decreasing function of care in both activities, and let the reaction in each activity be negative whether liable or not.   Then it is more efficient to have liability placed on activity one if any of the following sets of conditions is satisfied:

   (3.1)   Activity one is more effective in cost avoidance (i.e. (9) holds); B is a nonincreasing function of x; and the response of activity one is greater than one, either when liable or when not liable.

   (3.2)   Activity one is more effective in cost avoidance; B is a nonincreasing function of x; and, comparing either the liable reactions or the non-liable reactions, activity one has the greater excess supply of care (i.e. (22) or (23) holds).

   (3.3)   With liability placed on activity one, care in activity two has a positive effect on costs in activity one, and care in activity one has a negative effect on costs in activity two.

Conditions (3.1) and (3.2) come from our use of Lemma 1, while (3.3) comes from Lemma 2.

We have discussed only one among many possible cases.   These can be classified by the sign of the reactions in the two activities, and

by the signs of C_1 and C_2 which give the relative positions of the liable and non-liable reaction functions in the two activities. What we are looking for are conditions on the functions A, B, and C. In some of the 16 cases, it seems that no convenient assumptions give the desired result. In others the situation is simpler than the one described in Theorem 3. Rather than set out further theorems, we describe briefly some cases in sufficient detail to allow the reader to state and prove theorems if he so wishes. The basic nature of the type of information required has been indicated in the development of Theorem 3, but some cases suggest new forms for the conditions.

Negative Reactions, $C_1 > 0$, $C_2 < 0$

In contrast to the case discussed above, the x*-curve now lies to the left of the x'-curve. We can apply a variant of Lemma 2. (A + C nondecreasing in y and B nondecreasing in x. This is the external diseconomies case by and large.) For this we must ensure $y* \gtreqless y'$ and $x' \gtreqless x*$ as shown in Figure 2. This will also guarantee $x* - x' \lesseqgtr y* - y'$ and so permit use of the variant to Lemma 1 with B nondecreasing in x. To have $y* \gtreqless y'$, $y*(x) - y'(x)$ must be sufficiently large for the intersection of y' with · x* to lie below y*, i.e. for $y* \gtreqless y'(x*(y*))$. This will hold if and only if

$$\frac{y*(x') - y'(x')}{x'(y*) - x*(y*)} \geq \frac{y'(x*(y*)) - y'(x')}{x'(y*) - x*(y*).}$$

A sufficient condition for this is

$$\frac{y^*(x') - y'(x')}{x'(y^*) - x^*(y^*)} \geq \sup \left| \frac{dy'}{dx} \right|$$

since $y'(x^*(y^*)) - y'(x')$ is the integral of $\left| \frac{dy'}{dx} \right|$ over the range $x^*(y^*)$ to $x'(y^*)$.[15]

Positive Reactions, $C_1 < 0$, $C_2 < 0$

To obtain the results $y^* > y'$ and $x^* > x'$, as depicted in Figure 3, we can use conditions on the distances between reaction functions relative to their slopes. From the Figure we see $x^* \geq x'$ if $x'^{-1}(x^*) \geq y^*(x^*)$. Measuring from the point $(x'(y'), x')$ we can express this as $x^* \geq x'$ if and only if

$$\frac{x'^{-1}(x^*) - y'}{x^*(y') - x'(y')} \geq \frac{y^*(x^*) - y'(x^*)}{x^*(y') - x'(y')}.$$

Since

$$x'^{-1}(x^*) - y' = \int_{x'(y')}^{x^*(y')} \left(\frac{dx'(s)}{dy} \right)^{-1} ds,$$

a sufficient condition for $x^* \geq x'$ is

$$\frac{y^*(x^*) - y'(x^*)}{x^*(y') - x'(y')} \leq \left(\sup \left| \frac{dx'}{dy} \right| \right)^{-1}.$$

Similarly, a sufficient condition for $y^* \geq y'$ is

$$\frac{y^*(x^*) - y'(x^*)}{x^*(y') - x'(y')} \geq \sup \left| \frac{dy^*}{dx} \right|.$$

<u>Positive Reactions, $C_1 > 0$, $C_2 < 0$</u>

This case is particularly simple since without further assumptions $x^* - x' < 0 < y^* - y'$. (Cf. Figure 4.) Thus both Lemmas can be applied by satisfying the appropriate additional assumptions.

<u>Mixed Reactions, $C_1 < 0$, $C_2 > 0$</u>

For the case where x' and x^* increase in y while y^* and y' decrease in x we have the situation depicted in Figure 5. It is immediate from the diagram that $y^* \geq y'$. Since the line between the two equilibria lies between $y'(x)$ and $y^*(x)$ between x' and x^*, it follows that $y^* - y' \geq x^* - x'$ if either response in activity two is greater than one.

6. <u>Outside and Nontransferable Externalities</u>

In the previous section, the possibility of outside externalities, represented by the function D, has been ignored. We discuss briefly how D must be incorporated into the various conditions of section 5 if the efficient allocation of liability is to be determinate. Notice first that the discussion of the relative positions of the points (x^*, y') and (x', y^*) in the plane is unaffected by the presence of D, since they concern only the choices of participants in the activities, which are never affected by D. It is therefore only necessary to consider how D is to be incorporated into the two lemmas.

In the proof of Lemma 1, the argument will work provided that $B + D$ is a nonincreasing function of x (and $x^* - x' \geq y^* - y'$). This is the only change required.

In Lemma 2, however, cost-minimization in both activities plays an essential role in the argument. One way of extending the lemma is to replace condition (ii) by the asumption that B + D is non-increasing in x and x* ≥ x', and then to add a third condition: (iii) D is a nondecreasing function of y and y* ≥ y'. It is readily checked that the same form of proof now establishes the extended form of Lemma 2. Alternatively one could replace A + C by A + C + D in (i) and add (iii) D is nonincreasing in x and x* ≥ x'.

What started out as a fairly direct exploration of the implications of care in one activity being more important than care in the other ends in a maze of possible cases. The maze comes from the enormous variety that can be shown by equilibria with externalities when decisions in each activity affect decisions in the other activity. In some particular cases it may be more tractable to check the appropriate conditions from this catalogue than to estimate the two equilibrium positions and directly compare the total costs at the two equilibria. Alternatively the converse may often be true. Without the ability to determine the cheapest cost avoider, the notion cannot play a key role in legal-economic analysis. Unfortunately, this determination, apart from the simplest cases, appears more complicated than one might have hoped.

Appendix

In the text, the approach taken to the asymmetry of S was to measure differences in care between a given point and the point (x',y'), since (x',y') proves to be a convenient origin for the problem. However in some cases it may be very difficult to determine the values x' and y'. One way around this problem was noted in footnote 10, by identifying a much stronger condition which was sufficient to imply the asymmetry condition used. An alternative approach, taken here, is to examine conditions using asymmetry relative to an arbitrary point which serves as origin. This will be particularly appropriate when the care variables represent expenditures and are measured from the point of zero expenditures.

The alternative definition of asymmetry is

$$\text{For } s > t, \quad S(s,t) \lessgtr S(t,s). \tag{A.1}$$

(A.1) is implied by the condition in footnote 10, but is substantially weaker.[16] We present theorems which use this assumption among others to deduce the greater efficiency of assigning liability to activity one. As in the main text we have a simple result when all externalities are transferable.

Theorem 4   It is more efficient to have liability placed on activity
   one if A is independent of y, B of x, D = 0 and

   (i) $A'(s) \gtreqless B'(s)$ (all s)

   (ii) $C(x,y)$ is a decreasing function of y

   (iii) $S(s,t) \lesseqgtr S(t,s)$ (s > t).

Proof: (i) implies that $y' \geq x'$, for it implies that $A'(y') \geq 0$.
(ii) implies that $y^* > y'$. Therefore $y^* > x'$ and by (iii)

$$S(x',y^*) \geq S(y^*,x')$$

$$= A(y^*) + C(y^*,x') + B(x')$$

$$\geq A(y^*) + C(y^*,y') + B(y')$$

since $y' \geq x'$, and y' minimizes B. Now x^* minimizes
$A(x) + C(x,y')$. Therefore, we have

$$S(x',y^*) \geq S(x^*,y').//$$

When A depends on y, we have the same problem as before, that
the last step of the above proof does not go through. We proceed by
first establishing what inequalities between care levels can do for us.

Lemma 3   It is more efficient to have liability placed on activity one
if

     (i) A + C, D are nonincreasing in y, and $y' \geq x'$;

     (ii) B, D are nonincreasing in x, and $x^* \geq y^*$;   (A.2)

     (iii) $S(s,t) \leq S(t,s)$ when $s > t$, and $y^* \geq x'$.

Variants of Lemma 3 Any of the assumption pairs (i), (ii), or (iii)
can be changed to the opposite sense without affecting the conclusion.

Proof of Lemma 3

$S(x',y^*) \geq S(y^*,x')$ by (iii)

$$= \left[A(y^*,x') + C(y^*,x') \right] + B(y^*,x') + D(y^*,x')$$

$$\geq \left[A(y^*,y') + C(y^*,y') \right] + B(x^*,x') + D(x^*,y'), \quad \text{by (i) and (ii)}$$

$$\geq \left[A(x^*,y') + C(x^*,y') \right] + B(x^*,y'),$$

by the minimizing choices of the two activities. Therefore
$S(x',y*) \overset{\geq}{-} S(x*,y')$. //

It will be noticed that the Lemma and its proof are a kind of cross between Lemmas 1 and 2.

To obtain a satisfactory theorem, one must find conditions implying the inequalities between the x's and the y's. As in section 5, one has to consider different cases, depending on the signs of reactions and of C_1 and C_2. For illustration, we take the case of positive reactions, with the product of the responses less than one to ensure stability.

In Figure 6 we compare y' and x', and y* and x* by reflecting the x*- and y*-curves in the forty-five degree line. The reflected curves are shown as broken lines. It is clearly sufficient for

$$y' \overset{\geq}{-} x', \qquad x* \overset{\geq}{-} y*$$

that the curves are related as shown, i.e.

$$x'(s) \overset{\leq}{-} y'(s) \qquad\qquad (A.3)$$

and

$$y*(t) \overset{\leq}{-} x*(t). \qquad\qquad (A.4)$$

(A.3) is implied by the following condition on A and B:

$$B_2(s,t) \overset{\leq}{-} A_1(t,s); \qquad\qquad (A.5)$$

for $A_1(x'(t),t) = 0$, so that with (A.5) we have $B_2(t,x'(t)) \overset{\leq}{-} 0$, and since $B(t,y)$ is convex with minimum at $y = y'(t)$, $y'(t) \overset{\geq}{-} x'(t)$. Similarly, (A.4) follows from the condition

$$A_1(s,t) + C_1(s,t) \overset{<}{=} B_2(t,s) + C_2(t,s). \tag{A.6}$$

(A.5) and (A.6) specify certain asymmetries between the activities.

To establish $y* \overset{\geq}{=} x'$, the remaining inequality, we must find conditions that ensure the point $(x',y*)$ lies above the forty-five degree line. It is sufficient that

$$\text{For } s > t, \quad A_1(s,t) \overset{\geq}{=} B_2(t,s) + C_2(t,s). \tag{A.7}$$

This implies that for each t such that $x'(t) > t$,

$$y*(t) \overset{\geq}{=} x'(t). \tag{A.8}$$

In particular, if $x' = x'(y*) > y*$, and $t = y*$,

$$y*(t) < y*(x') = y* \tag{A.9}$$

since the $y*$-reaction is positive. Combining (A.8) and (A.9), we find that $x' > y*$ implies $x' \overset{<}{=} y*(t) < y*$, a contradiction. Therefore (A.7) implies $x' \overset{<}{=} y*$, as claimed.

We state these conclusions as

Theorem 5   Let all reactions be positive. It is more efficient to
     have liability placed on activity one if the following conditions
     hold.

     (i) $A + C$ and D are nonincreasing in y, and

$$A_1(s,t) \overset{\geq}{=} B_2(t,s); \tag{A.5}$$

     (ii) B and D are nonincreasing in x, and

$$A_1(s,t) + C_1(s,t) \overset{<}{=} B_2(t,s) + C_2(t,s); \tag{A.6}$$

(iii) $\qquad S(s,t) \lesseqgtr S(t,s) \qquad (s > t)$

and $\quad A_1(s,t) \gtreqless B_2(t,s) + C_2(t,s) \qquad (s > t).$ $\qquad$ (A.7)

<u>Corollary</u>. If $C_2 \lesseqgtr 0$, condition (A.7) is unnecessary (because it is implied by (A.5)).

Notice that $C_2 \lesseqgtr 0$ implies, by (A.5) and (A.6), that $C_1 \lesseqgtr 0.$

Footnotes

1.   [2].

2.   Op.cit., p. 136ff. In a more recent paper, Calabresi and Hirschoff, [3] have gone further in defining the cheapest cost avoider (p. 1060):

"which of the parties to the accident is in the best position to make the cost-benefit analysis between accident costs and accident avoidance costs and to act on that decision once it is made."

3.   One can be interested in efficiency without feeling that it alone should determine the assignment of liability. An alternative basis for assignment is advanced by Fletcher [6].

4. In practice, an exception to such a flat rate is generally made for intentional damage, as opposed to accidents.

5. See e.g. Brown [1], Diamond [4], [5], and Posner [7], [8], as well as Calabresi.

6. A strict liability rule is one which allocates costs to a party independent of particular behaviour giving rise to an accident. Such a rule has been applied in many particular settings. Specifically costs arising from accidents from hazardous activities like blasting are borne by person in the hazardous activity. See Prosser [9], Ch. 13.

7. p. 147.

8. The fact that individuals will try to avoid accidents independent of insurance coverage does not alter the conclusion here unless the nonfinancial motives giving rise to this care are, themselves, affected by the shifting of costs.

9. In a later paper, we do plan to deal with the case where insurance blunts some incentives, but not all, as where insurance costs do not depend on behaviour once engaged in an activity, but do depend on whether the activity is engaged in at all. An example might be automobile ownership in some circumstances or use for commuting.

10.   For an analysis of effectiveness using (0,0) as origin rather than (x',y') see the Appendix. One might like to check assumption (9) without estimating x' and y' (particularly in the cases to be considered later where A depends on y and B on x). (9) holds for _any_ (x',y') if and only if

$$S_1(x,y) \overset{\le}{=} S_2(x,y) \qquad (\text{all } x,y) \qquad\qquad (9')$$

Since

$$S_1 - S_2 = \underset{t \downarrow 0}{\text{Lim}} \frac{1}{t}\{S(x + t,y) - S(x,y)\} \qquad - \underset{t \downarrow 0}{\text{Lim}} \frac{1}{t}\{S(x,y + t) - S(x,y)\},$$

(9) certainly implies (9'). To prove the converse, one can calculate

$$S(x + t,y) - S(x,y + t) = \int_o^t \frac{d}{d\theta} S(x + \theta, y + t - \theta)\, d\theta$$

$$= \int_o^t \{S_1(x + \theta, y + t - \theta) - S_2(x + \theta, y + t - \theta)\} d\theta$$

$$\overset{\le}{=} 0 \quad \text{if (9') holds and } t > 0.$$

(9') is, it must be emphasized, a very stringent assumption, excluding in particular the interesting case where S has a global minimum at finite $(\hat{x},\hat{y})$ with contours surrounding that point.

11.   A similar result follows for the diseconomies case, reversing the sign of t in (9).

12. As an example that shows $D_1 < 0$ is insufficient, consider:

$A = \frac{1}{2}(x - 1)^2$, $B = \frac{1}{2}(y - 1)^2$, $C = 10 - 2x - y$, $D = 15 - x - 6y$.

For this case, $(x^*, y') = (3,1)$, $(x', y^*) = (1,2)$, $S(x^*, y') = 11$,

$S(x', y^*) = 8\frac{1}{2}$. (9) is clearly satisfied.

13. Lemma 2 is essentially inapplicable to the case where A is independent of y and B of x; for the first part of (i) is then $C_2 \gtrless 0$, and the second requires $C_2 \lessgtr 0$.

14. If $B_1 \gtrless 0$, the condition is that the response of activity two under one or other regime should be not greater / than one.

15. The case of negative reactions with $C_1 > 0$, $C_2 > 0$ is essentially the same as that of Theorem 3, with (x,y) replaced by $(-x, -y)$. All the results translate simply. In the same way, $C_1 < 0$, $C_2 > 0$ is essentially the same case as $C_1 > 0$, $C_2 < 0$; and similar transformations extend the results proved below for positive and mixed reactions.

16. In footnote 10 it was shown that (9') implies (9) for any (x',y'). Putting $(x',y') = (0,0)$, we have (A.1).
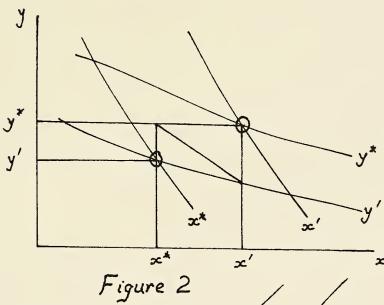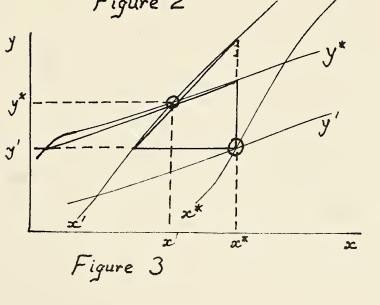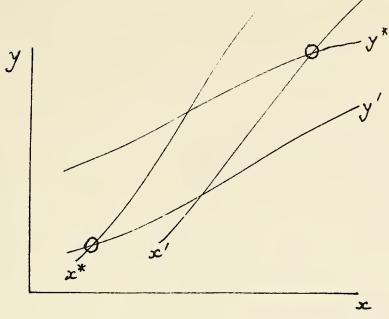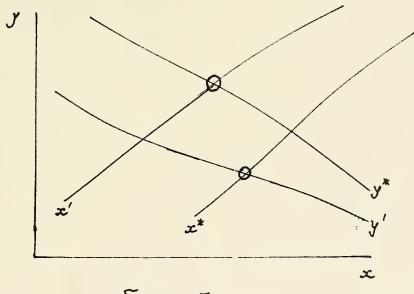
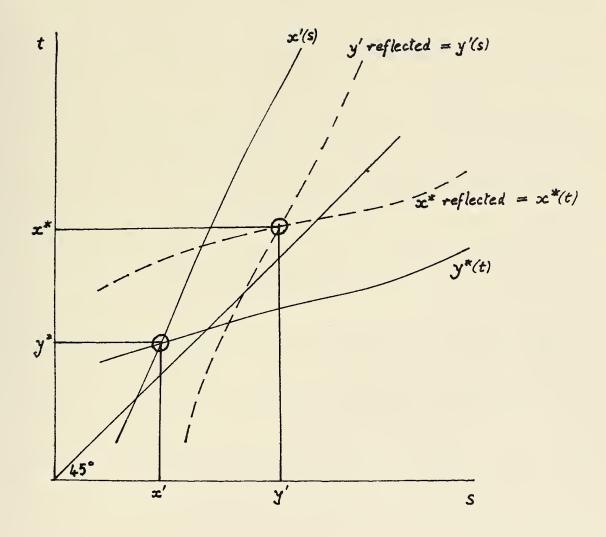Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

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BASEMENT

Date Due